$$\lim_{x \to \infty} \left(\sqrt{x(x+a)} - x \right) = \lim_{x \to \infty} \frac{\left(\sqrt{x(x+a)} - x \right)\left(\sqrt{x(x+a)} + x \right)}{\left(\sqrt{x(x+a)} + x \right)}$$

$$= \lim_{x \to \infty} \frac{x(x+a) - x^2}{\left(\sqrt{x(x+a)} + x \right)} = \lim_{x \to \infty} \frac{xa}{\left(\sqrt{x(x+a)} + x \right)} = \lim_{x \to \infty} \frac{xa}{\left(x\sqrt{1 + \dfrac{a}{x}} + x \right)}$$

$$= \lim_{x \to \infty} \frac{xa}{x\left(\sqrt{1 + \dfrac{a}{x}} + 1 \right)} = \lim_{x \to \infty} \frac{a}{\left(\sqrt{1 + \dfrac{a}{x}} + 1 \right)} = \frac{a}{2}$$