Chair of Statistics and Econometrics, RUB
Prof. V. Golosnoy, Dr. M. Seifert

Summer term 2017

Advanced Statistics – Exercise 5

Let $X_1, X_2, \dots, X_n$ be a random sample from a density f with mean μ and finite variance σ^2.

1. Consider the estimator $\overline{X} = \frac{1}{n}\sum\limits_{i=1}^{n} X_i$. Derive $E[\overline{X}]$ and $V[\overline{X}]$.

2. Consider the estimators

$$\begin{aligned} \hat{\sigma}_A^2 &= \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2 \\ \hat{\sigma}_B^2 &= \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2 \end{aligned}$$

   Derive $E[\hat{\sigma}_A^2]$ and $E[\hat{\sigma}_B^2]$.
   Is one of the estimators $\hat{\sigma}_A^2$ or $\hat{\sigma}_B^2$ an unbiased estimator for the variance σ^2 ?

3. Consider the random variable $Y_n = \frac{1}{\sqrt{n}\sigma}(\sum\limits_{i=1}^{n} X_i - n\mu)$.
   Show that the third moment $E(Y_n^3) \to 0$ as $n \to \infty$.